Entangling two-level atoms by spontaneous emission

L. Jakóbczyk

Institute of Theoretical Physics
University of Wrocław
Pl. M. Borna 9, 50-204 Wrocław, Poland

ABSTRACT: It is shown that the dissipation due to spontaneous emission can entangle two closely separated two-level atoms.

1 Introduction

Analysis of various aspects of spontaneous emission by a system of two-level atoms, initiated in the classical paper of Dicke [1] was further developed by several authors (see e.g. [2, 3, 4]). In particular, in the case of spontaneous emission by two trapped atoms separated by a distance small compared to the radiation wavelength, where is a substantial probability that a photon emitted by one atom will be absorbed by the other, there are states of the system in which photon exchange can enhance or diminish spontaneous decay rates. The states with enhanced decay rate are called superradiant and analogously states with diminished decay rate are called subradiant [1]. It was also shown by Dicke, that the system of two coupled two-level atoms can be treated as a single four-level system with modified decay rates. Note also that such model can be realized in a laboratory by two laser-cooled trapped ions, where the observation of superradiance and subradiance is possible [5].

Another aspects of the model of the spontaneous emission are studied in the present paper. When the compound system of two atoms is in an entangled state, the irreversible process of radiative decay usually destroys correlations and the state becomes unentangled. In the model studied here, the photon exchange produces correlations between atoms which can partially overcome decoherence caused by spontaneous radiation. As a result, some amount of entanglement can survive, and moreover there is a possibility that this process can entangle separable states of two atoms. The idea that dissipation can create entanglement in physical systems, was recently developed in several papers [6, 7, 8, 9]. In the present paper we show that the dissipation due to spontaneous emission can entangle two atoms that are initially prepared in a separable state. We study the dynamics of this process. In the Markovian approximation it is given by the semi-group $\{T_t\}$ of completely positive linear mappings acting on density matrices [10]. We consider time evolution of initial state of the system as well as the evolution of its entanglement, measured by so called concurrence [11, 12], in the case when the photon exchange rate γ is close to spontaneous emission rate of the single atom γ_0 and we can use the approximation $\gamma_0 = \gamma$ (similar model was also considered in [13]). We calculate asymptotic stationary states ρ_{as} for the semi-group $\{T_t\}$ and show that they depend on initial conditions (i.e. $\{T_t\}$ is relaxing but not uniquely relaxing). The concurrence of ρ_{as} also depends on initial state and can be non zero for some of them. We discuss in details some classes of initial states. In particular, we show that there are pure separable states evolving to entangled mixed states and such which remain separable during evolution. The first class contains physically interesting initial state when one atom in in excited state and the other is in ground state. The relaxation process given by the semi-group $\{T_t\}$ produces in this case the states with entanglement monotonically increasing in time to the maximal value. The class of pure maximally entangled initial states is also discussed. Similar "production" of entanglement is shown to be present for some classes of mixed states. On the other hand, when the photon exchange rate γ is smaller then γ_0, the relaxation process brings all initial states to the unique asymptotic state when both atoms are in its ground states. As we show, even in that case the dynamics can entangle two separable states, but the amount of entanglement is decreasing to zero.

2 Pair of two-level atoms

Consider two-level atom A with ground state $|0\rangle$ and excited state $|1\rangle$. This quantum system can be described in terms of the Hilbert space $\mathcal{H}_A = \mathbb{C}^2$ and the algebra $\mathfrak{A}_A$ of 2×2 complex matrices. If we identify $|1\rangle$ and $|0\rangle$ with vectors $\binom{1}{0}$ and $\binom{0}{1}$ respectively, then the raising and lowering operators $\sigma_+,\ \sigma_-$ defined by

$$\sigma_+ = |1\rangle\langle 0|, \quad \sigma_- = |0\rangle\langle 1| \tag{1}$$

can be expressed in terms of Pauli matrices $\sigma_1,\ \sigma_2$

$$\sigma_+ = \frac{1}{2}\,(\sigma_1 + i\,\sigma_2), \quad \sigma_- = \frac{1}{2}\,(\sigma_1 - i\,\sigma_2) \tag{2}$$

For a joint system AB of two two-level atoms A and B, the algebra $\mathfrak{A}_{AB}$ is equal to 4×4 complex matrices and the Hilbert space $\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B = \mathbb{C}^4$. Let $\mathcal{E}_{AB}$ be the set of all states of the compound system i.e.

$$\mathcal{E}_{AB} = \{\rho \in \mathfrak{A}_{AB}\,:\, \rho \geq 0 \quad \text{and} \quad \mathrm{tr}\,\rho = 1\} \tag{3}$$

The state $\rho \in \mathcal{E}_{AB}$ is *separable* [14], if it has the form

$$\rho = \sum_k \lambda_k \rho_k^A \otimes \rho_k^B, \quad \rho_k^A \in \mathcal{E}_A,\ \rho_k^B \in \mathcal{E}_B,\ \lambda_k \geq 0 \quad \text{and} \quad \sum_k \lambda_k = 1 \tag{4}$$

The set $\mathcal{E}_{AB}^{\,\mathrm{sep}}$ of all separable states forms a convex subset of $\mathcal{E}_{AB}$. When ρ is not separable, it is called *inseparable* or *entangled*. Thus

$$\mathcal{E}_{AB}^{\,\mathrm{ent}} = \mathcal{E}_{AB} \setminus \mathcal{E}_{AB}^{\,\mathrm{sep}} \tag{5}$$

If $P \in \mathcal{E}_{AB}$ is a pure state i.e. P is one-dimensional projector, then P is separable iff partial traces $\mathrm{tr}_A P$ and $\mathrm{tr}_B P$ are also projectors. For mixed states, the separability problem is much more involved. Fortunately, in the case of $4-$level compound system there is a simple necessary and sufficient condition for separability: ρ is separable iff its partial transposition ρ^{T_A} is also a state [15]. Another interesting question is how to measure the amount of entanglement a given quantum state contains. For a pure state P, the entropy of entanglement

$$E(P) = -\mathrm{tr}\,[(\mathrm{tr}_A P)\,\log_2\,(\mathrm{tr}_A P)] \tag{6}$$

is essentailly a unique measure of entanglement [16]. For mixed state ρ it seems that the basic measure of entanglement is the entanglement of formation [17]

$$E(\rho) = \min \sum_k \lambda_k E(P_k) \tag{7}$$

where the minimum is taken over all possible decompositions

$$\rho = \sum_k \lambda_k P_k \tag{8}$$

Again, in the case of 4 – level system, $E(\rho)$ can be explicitely computed and it turns out that $E(\rho)$ is the function of another useful quantity $C(\rho)$ called *concurrence*, which also can be taken as a measure of entanglement [11, 12]. Since in the paper we use concurrence to quantify entanglement, now we discuss its definition. Let

$$\rho^{\dagger} = (\sigma_2 \otimes \sigma_2)\, \overline{\rho}\, (\sigma_2 \otimes \sigma_2) \tag{9}$$

where $\overline{\rho}$ is the complex conjugation of the matrix ρ. Define also

$$\widehat{\rho} = (\rho^{1/2} \rho^{\dagger} \rho^{1/2})^{1/2} \tag{10}$$

Then the concurrence $C(\rho)$ is given by [11, 12]

$$C(\rho) = \max\,(\,0, 2p_{\max}(\widehat{\rho}) - \mathrm{tr}\,\widehat{\rho}\,) \tag{11}$$

where $p_{\max}(\widehat{\rho})$ denotes the maximal eigenvalue of $\widehat{\rho}$. The value of the number $C(\rho)$ varies from 0 for separable states, to 1 for maximally entangled pure states.

3 Decay in a system of closely separated atoms

We study the spontaneous emission of two atoms separated by a distance R small compared to the radiation wavelength . At such distances there is a substantial probability that the photon emitted by one atom will be absorbed by the other. Thus the dynamics of the system is given by the master equation [18]

$$\frac{d\rho}{dt} = L\rho, \quad \rho \in \mathcal{E}_{AB} \tag{12}$$

with the following generator L

$$\begin{aligned} L\rho =& \frac{\gamma_0}{2}\,[2\sigma_-^A\,\rho\,\sigma_+^A + 2\sigma_-^B\,\rho\,\sigma_+^B - (\sigma_+^A\,\sigma_-^A + \sigma_+^B\,\sigma_-^B)\,\rho - \rho\,(\sigma_+^A\,\sigma_-^A + \sigma_+^B\,\sigma_-^B)] + \\ & \frac{\gamma}{2}\,[2\sigma_-^A\,\rho\,\sigma_+^B + 2\sigma_-^B\,\rho\,\sigma_+^A - (\sigma_+^A\,\sigma_-^B + \sigma_+^B\,\sigma_-^A)\,\rho - \rho\,(\sigma_+^A\,\sigma_-^B + \sigma_+^B\,\sigma_-^A)] \end{aligned} \tag{13}$$

where

$$\sigma_\pm^A = \sigma_\pm \otimes \mathbb{I},\; \sigma_\pm^B = \mathbb{I} \otimes \sigma_\pm,\; \sigma_\pm = \frac{1}{2}(\sigma_1 \pm i\sigma_2) \tag{14}$$

Here γ_0 is the single atom spontaneous emission rate, and $\gamma = g\gamma_0$ is a relaxation constant of photon exchange. In the model, g is the function of the distance R between atoms and $g \to 1$ when $R \to 0$. In this section we investigate the time evolution of the initial density matrix ρ of the compound system, governed by the semi - group $\{T_t\}_{t\geq 0}$ generated by L. In particular, we will study the time development of entanglement of ρ, measured by concurrence.

Assume that the distance between atoms is so small that the exchange rate γ is close to γ_0 and we can use the approximation $g = 1$. Under this condition we study evolution of the system and in particular we consider asymptotic states. Direct calculations show that the semi - group $\{T_t\}$ generated by L with $g = 1$ is relaxing but not uniquely relaxing i.e. there are as many stationary states as there are initial conditions. More precisely, for a given initial state $\rho = (\rho_{jk})$, the state

$\rho(t)$ at time t has the following matrix elements

$$\rho_{11}(t) = e^{-2\gamma_0 t}\rho_{11}$$
$$\rho_{12}(t) = \frac{1}{2}[e^{-2\gamma_0 t}(\rho_{12} + \rho_{13}) + e^{-\gamma_0 t}(\rho_{12} - \rho_{13})]$$
$$\rho_{13}(t) = \frac{1}{2}[e^{-2\gamma_0 t}(\rho_{12} + \rho_{13}) + e^{-\gamma_0 t}(\rho_{13} - \rho_{12})]$$
$$\rho_{14}(t) = e^{-\gamma_0 t}\rho_{14}$$
$$\rho_{22}(t) = \frac{1}{4}e^{-2\gamma_0 t}(\rho_{22} + \rho_{33} + 2\mathrm{Re}\,\rho_{23}) + \frac{1}{2}e^{-\gamma_0 t}(\rho_{22} - \rho_{33}) + \gamma_0 t e^{-2\gamma_0 t}\rho_{11} + \frac{1}{4}(\rho_{22} + \rho_{33} - 2\mathrm{Re}\,\rho_{23})$$
$$\rho_{23}(t) = \frac{1}{4}e^{-2\gamma_0 t}(\rho_{22} + \rho_{33} + 2\mathrm{Re}\,\rho_{23}) + \frac{1}{2}e^{-\gamma_0 t}(\rho_{23} - \rho_{32}) + \gamma_0 t e^{-2\gamma_0 t}\rho_{11} - \frac{1}{4}(\rho_{22} + \rho_{33} - 2\mathrm{Re}\,\rho_{23})$$
$$\rho_{24}(t) = -\frac{1}{2}e^{-2\gamma_0 t}(\rho_{12} + \rho_{13}) + \frac{1}{2}e^{-\gamma_0 t}(2\rho_{12} + 2\rho_{13} + \rho_{24} + \rho_{34}) + \frac{1}{2}(\rho_{24} - \rho_{34})$$
$$\rho_{33}(t) = \rho_{22}(t)$$
$$\rho_{34}(t) = -\frac{1}{2}e^{-2\gamma_0 t}(\rho_{12} + \rho_{13}) + \frac{1}{2}e^{-\gamma_0 t}(2\rho_{12} + 2\rho_{13} + \rho_{24} + \rho_{34}) - \frac{1}{2}(\rho_{24} - \rho_{34})$$
$$\rho_{44}(t) = -e^{-2\gamma_0 t}(\rho_{11} + \rho_{22} + \mathrm{Re}\,\rho_{23}) - 2\gamma_0 t e^{-2\gamma_0 t}\rho_{11} + \frac{1}{2}(1 + \rho_{11} + \rho_{44} + 2\mathrm{Re}\,\rho_{23})$$

and remaining matrix elements can be obtained by hermiticity condition $\rho_{kj} = \overline{\rho}_{jk}$. In the limit $t \to \infty$ we obtain asymptotic (stationary) states parametrized as follows

$$\rho_{\mathrm{as}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \alpha & -\alpha & \beta \\ 0 & -\alpha & \alpha & -\beta \\ 0 & \overline{\beta} & -\overline{\beta} & 1-2\alpha \end{pmatrix} \tag{15}$$

where

$$\alpha = \frac{1}{4}(\rho_{22} + \rho_{33} - 2\mathrm{Re}\,\rho_{23}), \quad \beta = \frac{1}{2}(\rho_{24} - \rho_{34}) \tag{16}$$

We can also compute concurrence of the asymptotic state and the result is:

Concurrence of asymptotic state of the semi - group $\{T_t\}$ generated by L with $g = 1$ equals to

$$C(\rho_{\mathrm{as}}) = 2|\alpha| = \frac{1}{2}|\rho_{22} + \rho_{33} - 2\mathrm{Re}\,\rho_{23}| \tag{17}$$

where ρ_{jk} are the matrix elements of the initial state.

4 Some examples

In this section we consider examples of initial states and its evolution.

I. Pure separable states

Let

$$\rho = P_{\Psi\otimes\Phi} = P_\Psi \otimes P_\Phi \tag{18}$$

where

$$\Psi = \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix} \in \mathcal{H}_A, \quad \Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \in \mathcal{H}_B$$

are normalized. Then one can check that

$$
\begin{aligned}
\alpha &= \frac{1}{4}(1 - |\langle\, \Psi,\, \Phi\, \rangle|^2) \\
\beta &= \frac{1}{2}(|\Phi_2|^2 \Psi_1 \overline{\Psi}_2 - |\Psi_2|^2 \Phi_1 \overline{\Phi}_2)
\end{aligned}
\tag{19}
$$

where $\langle\, \cdot,\, \cdot\, \rangle$ is the inner product in $\mathbb{C}^2$. So

$$
C(\rho_{\mathrm{as}}) = \frac{1}{2}(1 - |\langle\, \Psi,\, \Phi\, \rangle|^2) \tag{20}
$$

From the formula (21) we see that there are separable initial states for which asymptotic states are entangled. In particular, the asymptotic state has a maximal concurrence if vectors Ψ and Φ are orthogonal and its concurrence is zero (the state remains separable) if $|\langle\, \Psi,\, \Phi\, \rangle| = 1$.
Now we discuss some special cases.
a. When one atom is in excited state and the other is in ground state

$$
\Psi = |1\rangle, \quad \Phi = |0\rangle
$$

the asymptotic (mixed) state is given by

$$
\rho_{\mathrm{as}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & -\frac{1}{4} & 0 \\ 0 & -\frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}
$$

It can also be shown that in this case the relaxation process produces the state ρ_t with concurrence

$$
C(\rho_t) = \frac{1 - e^{-\gamma_0 t}}{2}
$$

increasing to the maximal value equal to 1/2. Thus two atoms initially in separable state become entangled for all t and the asymptotic (steady) state attains the maximal amount of entanglement.
b. When two atoms are in excited states

$$
\Psi = \Phi = |1\rangle
$$

the asymptotic state equals to

$$
|0\rangle \otimes |0\rangle
$$

Thus the relaxation process brings two atoms into ground states.
c. The state $|0\rangle \otimes |0\rangle$ is stationary state for semi - group $\{T_t\}$.

II. Pure maximally entangled states

Let

$$
\rho = Q(a, \theta_1, \theta_2) = \begin{pmatrix}
\frac{a^2}{2} & \frac{a\sqrt{1-a^2}}{2} e^{-i\theta_1} & \frac{a\sqrt{1-a^2}}{2} e^{-i\theta_2} & -\frac{a^2}{2} e^{-i(\theta_1+\theta_2)} \\
\frac{a\sqrt{1-a^2}}{2} e^{i\theta_1} & \frac{1-a^2}{2} & \frac{1-a^2}{2} e^{i(\theta_1-\theta_2)} & -\frac{a\sqrt{1-a^2}}{2} e^{-i\theta_2} \\
\frac{a\sqrt{1-a^2}}{2} e^{i\theta_2} & \frac{1-a^2}{2} e^{-i(\theta_1-\theta_2)} & \frac{1-a^2}{2} & -\frac{a\sqrt{1-a^2}}{2} e^{-i\theta_1} \\
-\frac{a^2}{2} e^{i(\theta_1+\theta_2)} & -\frac{\sqrt{1-a^2}}{2} e^{i\theta_2} & -\frac{a\sqrt{1-a^2}}{2} e^{i\theta_1} & \frac{a^2}{2}
\end{pmatrix}
$$

where $a \in [0,1]$, $\theta_1, \theta_2 \in [0, 2\pi]$. Pure states $Q(a, \theta_1, \theta_2)$ are maximally entangled and form a family of all maximally entangled states of the 4 - level system [19]. It turns out that $\rho_{\rm as}$ is defined by

$$\begin{aligned} \alpha &= \frac{1}{4}(1-a^2)(1-\cos(\theta_1-\theta_2)) \\ \beta &= \frac{1}{4}a\sqrt{1-a^2}(e^{-i\theta_1}-e^{-i\theta_2}) \end{aligned} \tag{21}$$

and

$$C(\rho_{\rm as}) = \frac{1}{2}(1-a^2)(1-\cos(\theta_1-\theta_2)) \tag{22}$$

From the formula (23) we see that there are initial maximally entangled states which asymptotically become separable ($a = 1$ or $\theta_1 - \theta_2 = 2k\pi$) and such that the asymptotic concurrence is greater then 0. States with $a = 0$ and $\theta_1 - \theta_2 = (2k+1)\pi$ remain maximally entangled. For example the state

$$\frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle - |1\rangle \otimes |0\rangle) \tag{23}$$

is stable. On the other hand, the concurrence of

$$\frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle) \tag{24}$$

goes to zero faster then the concurrence of

$$\frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle) \tag{25}$$

as shown on **Fig. 1.** below. In Dicke's theory of spontaneous radiation processes the state (24) is called subradiant whereas the state (25) has half the lifetime of a single atom and therefore is called superradiant [1]. We see that the time-dependence of concurrence reflects the relaxation properties of those states.

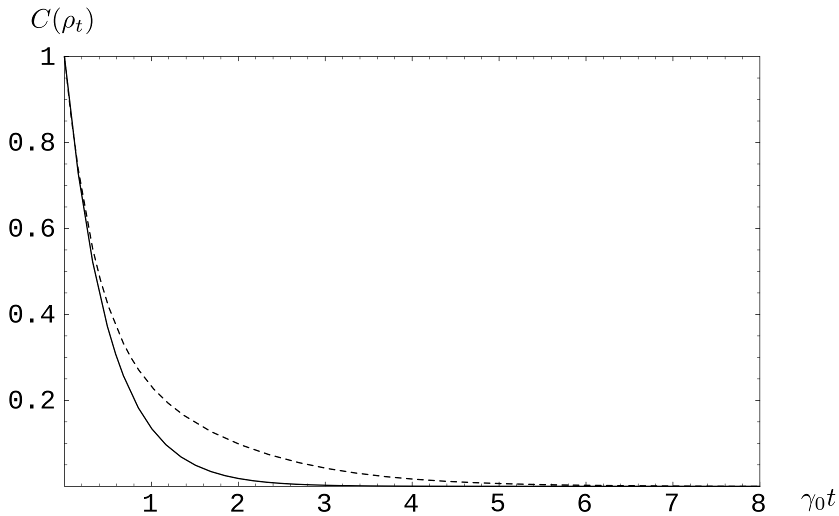

Fig. 1. Concurrence as the function of time for initial states: $\frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle)$ (dotted line) and $\frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle + |1\rangle \otimes |0\rangle)$ (solid line).

III. Some classes of mixed states

a. Bell - diagonal states. Let

$$\rho_{\mathrm{B}} = p_1|\Phi^+\rangle\langle\Phi^+| + p_2|\Phi^-\rangle\langle\Phi^-| + p_3|\Psi^+\rangle\langle\Psi^+| + p_4|\Psi^-\rangle\langle\Psi^-| \tag{26}$$

where Bell states $\Phi^\pm$ and $\Psi^\pm$ are given by

$$\Phi^\pm = \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle \pm |1\rangle \otimes |1\rangle), \quad \Psi^\pm = \frac{1}{\sqrt{2}}(|1\rangle \otimes |0\rangle \pm |0\rangle \otimes |1\rangle) \tag{27}$$

It is known that all $p_i \in [0, 1/2]$, ρ_{B} is separable, while for $p_1 > 1/2$, ρ_{B} is entangled with concurrence equal to $2p_1 - 1$ (similarly for p_2, $p_3\, p_4$) [20]. Now the asymptotic state has the form

$$\rho_{\mathrm{as}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{p_4}{2} & -\frac{p_4}{2} & 0 \\ 0 & -\frac{p_4}{2} & \frac{p_4}{2} & 0 \\ 0 & 0 & 0 & 1 - p_4 \end{pmatrix} \tag{28}$$

with concurrence $C(\rho_{\mathrm{as}}) = p_4$. So even when the initial state is separable, the asymptotic state becomes entangled.

b. Werner states [21]. Let

$$\rho_{\mathrm{W}} = (1 - p)\frac{\mathbb{I}_4}{4} + p|\Phi^+\rangle\langle\Phi^+| \tag{29}$$

If $p > 1/3$, ρ_{W} is entangled with concurrence equal to $(3p-1)/2$. On the other hand

$$\rho_{\mathrm{as}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1-p}{8} & \frac{p-1}{8} & 0 \\ 0 & \frac{p-1}{8} & \frac{1-p}{8} & 0 \\ 0 & 0 & 0 & \frac{3+p}{4} \end{pmatrix} \tag{30}$$

has the concurrence $C(\rho_{\mathrm{as}}) = \frac{1-p}{4}$, so the asymptotic states are entangled for all $p \neq 1$. Notice that even completely mixed state $\frac{\mathbb{I}_4}{4}$ evolves to entangled asymptotic state.

c. Maximally entangled mixed states . The states

$$\rho_{\mathrm{M}} = \begin{pmatrix} h(\delta) & 0 & 0 & \delta/2 \\ 0 & 1-2h(\delta) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \delta/2 & 0 & 0 & h(\delta) \end{pmatrix}, \quad h(\delta) = \begin{cases} 1/3 & \delta \in [0, 2/3] \\ \delta/2 & \delta \in [2/3, 1] \end{cases} \tag{31}$$

are conjectured to be maximally entangled for a given degree of inpurity measured by $\operatorname{tr} \rho^2$ [22]. According to (18) the concurrence of the asymptotic state is given by

$$C(\rho_{\mathrm{as}}) = \frac{1}{2}(1 - 2h(\delta)) \tag{32}$$

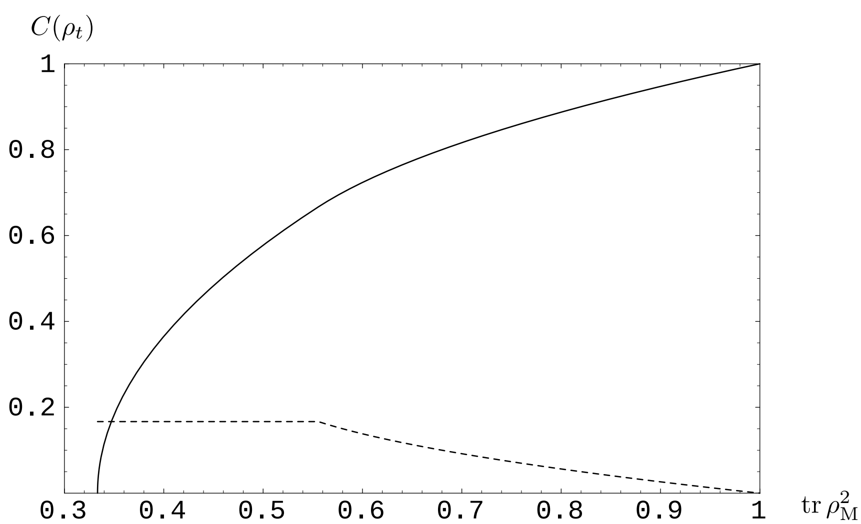

Fig. 2. Concurrence of ρ_{M} (solid line) and ρ_{as} (dotted line) as the function of $\operatorname{tr} \rho_{\mathrm{M}}^2$

Even in that case, there are initial states (for sufficiently small $\operatorname{tr} \rho_{\mathrm{M}}^2$) such that the asymptotic state is more entangled (see **Fig. 2.**).

5 Remarks on general case

In the case of arbitrary distance between the atoms i.e. when $g \in [0,1)$, semi-group generated by L is uniquely relaxing, with the asymptotic state $|0\rangle \otimes |0\rangle$. Thus, for any initial state ρ, the concurrence $C(\rho_t)$ approaches 0 when $t \to \infty$. But it does not mean that the function $t \to C(\rho_t)$ is always monotonic. The general form of $C(\rho_t)$ is rather involved, so we consider only some special cases.

1. Let the initial state of the compound system be equal to $|0\rangle \otimes |1\rangle$. This states evolves to

$$\rho_t = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2}e^{-\gamma_0 t}(\cosh\gamma t + 1) & -\frac{1}{2}e^{-\gamma_0 t}\sinh\gamma t & 0 \\ 0 & -\frac{1}{2}e^{-\gamma_0 t}\sinh\gamma t & \frac{1}{2}e^{-\gamma_0 t}(\cosh\gamma t - 1) & 0 \\ 0 & 0 & 0 & 1 - e^{-\gamma_0 t}\cosh\gamma t \end{pmatrix} \tag{33}$$

with concurrence

$$C(\rho_t) = e^{-\gamma_0 t}\sinh\gamma t \tag{34}$$

In the interval $[0, t_\gamma]$, where

$$t_\gamma = \frac{1}{2\gamma}\ln\frac{\gamma_0 + \gamma}{\gamma_0 - \gamma}$$

the function (34) is increasing to its maximal value

$$C_{\max} = \frac{\gamma}{\gamma_0 - \gamma}\left(\frac{\gamma_0 + \gamma}{\gamma_0 - \gamma}\right)^{-\frac{\gamma_0 + \gamma}{2\gamma}}$$

whereas for $t > t_\gamma$, $C(\rho_t)$ decreases to 0. Thus for any nonzero photon exchange rate γ, dynamics given by the semi - group $\{T_t\}$ produces some amount of entanglement between two atoms which are initially in the ground state and excited state. Note that the maximal value of $C(\rho_t)$ depends only on emission rates γ_0 and γ.

2. For the initial states

$$\Psi^{\pm} = \frac{1}{\sqrt{2}}\left(|0\rangle \otimes |1\rangle \pm |1\rangle \otimes |0\rangle\right)$$

the relaxation to the asymptotic state $|0\rangle \otimes |0\rangle$ is given by density matrices

$$\rho_t^{\pm} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2}e^{-(\gamma_0 \pm \gamma)t} & -\frac{1}{2}e^{-(\gamma_0 \pm \gamma)t} & 0 \\ 0 & -\frac{1}{2}e^{-(\gamma_0 \pm \gamma)t)} & \frac{1}{2}e^{-(\gamma_0 \pm \gamma)t} & 0 \\ 0 & 0 & 0 & 1 - e^{-(\gamma_0 \pm \gamma)t} \end{pmatrix} \tag{35}$$

with the corresponding concurrence

$$C(\rho_t^{\pm}) = e^{-(\gamma_0 \pm \gamma)t}$$

The state Ψ^- is no longer stable (as in the case of $\gamma = \gamma_0$), but during the evolution its concurrence goes to zero slower than $C(\rho_t^+)$ (**Fig. 3.**). For γ close to γ_0, Ψ^- is almost stable.

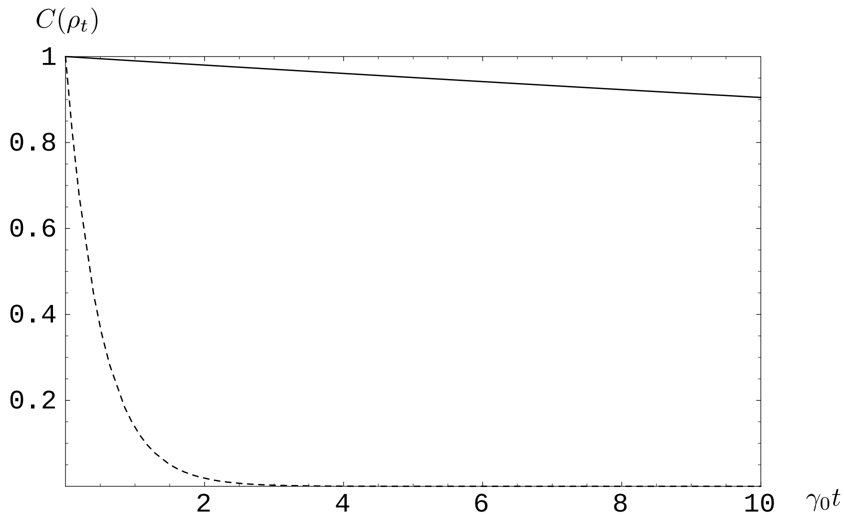

Fig. 3. $C(\rho_t^+)$ (dotted line) and $C(\rho_t^-)$ (solid line) for $\gamma/\gamma_0 = 0.99$

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